COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-62/99

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Nota interna: n° 169
26 Novembre 1962

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

ON THE DECAY OF THE η MESON.

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Abstract:

The decay of the η meson is estimated by considering higher mass contributions beside the ρ and ω. It is shown that the branching ratio $P(\eta \rightarrow 2\gamma)/P(\eta \rightarrow \pi^+\pi^-\pi^0)$ comes out well while we get a little larger ratio for $P(\eta \rightarrow \pi^+\pi^-\gamma)/P(\eta \rightarrow 2\gamma)$, although it may not be much in disagreement with present data.

(x) - On leave of absence from Nagoya University (Japan).

1) Since the discovery of the spin-parity 1^- octet, many phenomena have been explained by the use of pole diagrams due to these particles. Particulary the branching ratio

$$P(\eta \to 2\gamma)/P(\eta \to \pi^+\pi^-\gamma)$$

for 0^{-+} η meson was estimated by Gell Mann et al[1] provided that these decays go through a ρ pole diagram.

Neverthless the application of the same idea to $\eta \to 3\pi$ decay met with difficulty due to the closed loop integration. Beside the divergence problem, there seems to remain the difficulty of getting the observed ratio ~ 3 for Γ ($\eta \to$ neutral)/P($\eta \to \pi^+\pi^-\pi^0$) because of the smallness of the three body phase volume.

Recently, however, a mechanism which leads to suppression for $\pi^0 \to 2\gamma$ decay was analyzed by Geffen[2]. He obtains this result by taking into account higher mass contributions, beside those of ρ and ω, as suggested by the experimental form factor of $\pi^0 \to \gamma e^+e^-$. According to the idea of unitary symmetry, then, one would expect the same cancellation for $\eta \to 2\gamma$ decay. This fact indicates that the estimate of the ratio $\Gamma(\eta \to 2\gamma)$/P($\eta \to \pi^+\pi^-\pi^0$), based on the simple phase volume arguments, is no longer adeguate. To make it clear, we have to see how the same mechanism works on the $\eta \to 3\pi$ decay.

In this regard, it was pointed out by several people[3] that there is a similarity of the Dalitz plots for τ' and η decays. A reasonable explanation was given by saying that these decay modes are intermediated by the following steps,

$K^+ \to \pi^+ \to \pi^0\pi^0\pi^+$ and $\eta \to \pi_0 \to \pi^+\pi^-\pi^0$,

Of these steps the first one is a $\Delta I = \frac{1}{2}$ transition for K^+ and $\Delta I = 1$ e.m. transition for $0^{-+}\eta$. Then the isobaric spin of the final state is $I = 1$ for both cases and, as these cond step is due to the strong interaction, the Dalitz plots will be expected to be similar.

Based on this argument, we shall assume that the η-π^0 transition is the dominant contribution for $\eta \to 3\pi$ decay. As we will see later, this assumption leads to such a unique choice that we can conclude that the higher mass states, mentioned, should behave as a vector singlet state.

Several consequences which follow from the η-π transition mechanism, together with the contribution from the higher vector singlet state, are examined in this paper.

2) Let us define the vertex function between the P.S. state, a, and two vector states, b, c, as follows,

$$(1) \qquad \Gamma_{\mu\nu}^{abc}(k,k') \equiv \langle c_\nu(k') | j_\mu^{(b)}(0) | a(k+k') \rangle = \varepsilon_{\mu\nu\lambda\tau} k_\lambda k'_\tau F^{abc}(k^2)/m_\pi$$

where $j_\mu^{(b)}$ is the current for the b-meson. The γ_ρ and γ_ω are used hereafter as the renormalized coupling constants which appear in the currents, $j_\mu^{(\rho)}$, $j_\mu^{(\omega)}$ in a similar way as e does in $j_\mu^{(\gamma)}$ (4).

If the ρ, ω contribution is taken to be important the form factor for $\pi^0 \to \gamma\, e^+e^-$ is given by the formula,

$$(2) \qquad F^{\pi^0\gamma\gamma}(k^2) = f_{\pi_0\rho\omega} \frac{e^2}{2\sqrt{3}} \frac{1}{\gamma_\rho \gamma_\omega} \left(\frac{m_\rho^2}{m_\rho^2+k^2} + \xi \right)$$

where we put $F^{\pi^0\rho\omega}(k^2) = F^{\pi^0\omega\rho}(k^2) = f_{\pi_0\rho\omega}$ and $m_\rho = m_\omega$. A parameter ξ is inserted by Geffen(2) in order to take into account the higher mass contributions.

Instead of eq. (2), however, let us assume in this paper that the higher mass contributions can be represented by a state with a mass m_X. Since this state transforms into a γ, it may be natural to take it as a vector state. According to the unitary symmetry, we have several sets of states, namely the vector singlet and the vector octet etc.

As we will see later, the vector octet intermediate states however, give no $\eta-\pi$ transition in the limit of unitary symmetry. Thus, confining ourselves to the lower configuration, the representative state for the higher mass contribution should be taken as a vector singlet, if we insist on the importance of the $\eta-\pi$ transition.

In this case, $F^{\pi_0\gamma\gamma}(k^2)$ is defined, instead of by eq. (2), by

$$F^{\pi\gamma\gamma}(k^2) = e^2\left(\frac{f_{\pi\rho\omega}}{2\sqrt{3}\gamma_\omega} + \frac{f_{\pi\rho X}}{2\sqrt{3}\gamma_X}\right)\left(\frac{m_\rho^2}{m_\rho^2+k^2}\,\frac{1}{2\gamma_\rho}\right) +$$

$$(3) \qquad + \frac{1}{2\gamma_\rho}\left(\frac{f_{\pi\rho\omega}}{2\sqrt{3}\gamma_\omega}\,\frac{m_\omega^2}{m_\omega^2+k^2} + \frac{f_{\pi\rho X}}{2\sqrt{3}\gamma_X}\,\frac{m_X^2}{m_X^2+k^2}\right) =$$

$$= \frac{f_{\pi\omega\rho}\,e^2}{4\sqrt{3}\gamma_\omega\gamma_\rho}\left[(2+\xi)\frac{m_\rho^2}{m_\rho^2+k^2} + \xi\,\frac{m_X^2}{m_X^2+k^2}\right]$$

where

$$(4) \qquad \xi = \left(\frac{f_{\pi\rho X}}{\gamma_X}\right)\left(\frac{f_{\pi\rho\omega}}{\gamma_\omega}\right)^{-1},$$

Comparing eq. (3) with the experimental form factor for $\pi^0 \to \gamma e^+e^-$,

$$(5) \qquad F_{exp}^{\pi^0\gamma\gamma}(k^2) = F^{\pi^0\gamma\gamma}(0)\left[1 + a k^2/m_\pi^2\right],$$

$$a = -0.24 \pm 0.16,$$

we get

(6) $\xi = -1.12$ for $m_X \sim 2m_p$ and $a = -0.1$

The smaller m_x and the larger $|a|$ will bring ξ to be very near to -1 which will not be consistent with the $\omega \to 3\pi$ decay rate[(x)].

The $\pi^o \to 2\gamma$ decay rate is given by

$$\Gamma(\pi^o \to 2\gamma) = \frac{[F^{\pi\gamma\gamma}(0)]^2}{64\pi} m_\pi \sim 3 \times 10^{-6}\ \text{MeV} \tag{7}$$

and it follows from eq. (7) that

$$F^{\pi^o\gamma\gamma}(0) = (4\pi \times 0.7 \times 10^{-2})^{\frac{1}{2}} \alpha . \tag{8}$$

Comparing eq. (8) with eq. (3), we can estimate $f_{\pi\rho\omega}$ as

$$\frac{f^2_{\pi\rho\omega}}{4\sqrt{3}\,\gamma_\rho\gamma_\omega} = \frac{F^{\pi\gamma\gamma}(0)}{\alpha} \frac{\frac{\gamma_\rho^2\gamma_\omega^2}{(4\pi)^2}}{4\sqrt{3}\,\gamma_\rho\gamma_\omega} (2\sqrt{3})^2 \left(\frac{1}{1+\xi}\right)^2 = 0.43. \tag{9}$$

where we put $\gamma_\rho^2/4\pi \simeq \gamma_\omega^2/4\pi \simeq 1/2$. The $f_{\pi\rho\omega}$ calculated in this way is nine times larger than that calculated by Gell--Mann et al[(1)] in the case $\xi = 0$.

It is easy to see at this point that the experiment (5) suggests really the occurence of the cancellation between contributions from ρ, ω and x. Then the unitary symmetry[(5)] predicts the same cancellation for $\eta \to 2\gamma$.

Namely we have

$$F^{\eta\gamma\gamma}(0) = \frac{2e^2 f_{\eta\rho\rho}}{4\gamma_\rho^2} \left[1 - \frac{1}{3}(1 + 2\xi')\right] \tag{10}$$

where

$$2\xi' \equiv \left(\frac{f_{\eta\omega x}}{\gamma_x}\right)\left(\frac{f_{\eta\omega\omega}}{\gamma_\omega}\right)^{-1} .$$

(x) - Our choice eq. (6) gives $\Gamma(\omega \to 3\pi) = 9.4$ MeV.

Since the unitary symmetry gives the equalities,

(11) $f_{\eta\rho\chi} = f_{\eta\omega\chi} = f_{\pi\rho\chi}$ and $f_{\eta\omega\omega} = -f_{\eta\rho\rho} = -\frac{1}{2} f_{\pi\rho\omega}$,

we get in this limit $\xi' = -\xi$.

The decay rate, $P(\eta \to 2\gamma)$, depends on the value ξ' strongly. Thus we shall estimate it for two cases, one for $\xi' = -\xi$ and the other for $\xi' = 1.5$ which means that we consider a small deviation from eq. (11). Then we have

$$P(\eta\to 2\gamma) = \left(\frac{\sqrt{3}}{2}\right)^2 \left[\frac{1-\frac{1}{3}(1+2\xi')}{1+\xi}\right]^2 \left(\frac{m_\eta}{m_\pi}\right)^3 \Gamma(\pi^0 \to 2\gamma) =$$

(12) $= 0.64 \times 10^{-4}$ MeV for $\xi' = -\xi = 1.12$

$= 10.8 \times 10^{-4}$ MeV for $\xi' = 1.5$

3) The diagrams for $\eta \to 3\pi$ decay through ρ, ω intermediate states is given in Fig. 1 where the loops contain $\rho\gamma$ and $\omega\gamma$ e.m. transitions as is illustrated in Fig. 1a.

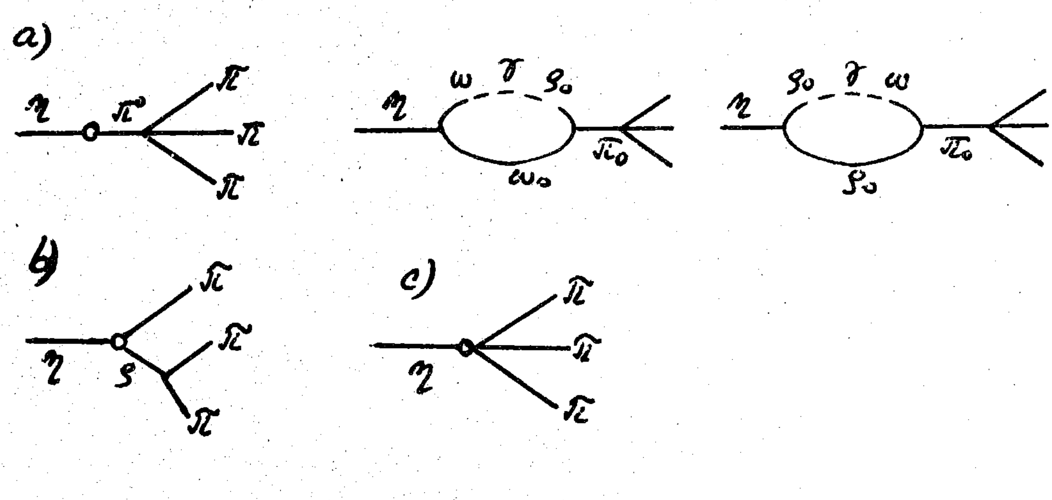

FIG. 1

Based on the Dalitz plot argument, as we discussed in the introduction, we simply disregard the diagrams in Fig. 1(b) and Fig. 1(c). But this is not valid if we consider only contributions from ρ , ω , because the diagrams in Fig. 1a are zero in the limit of unitary symmetry. This is easily seen from the relations in eq. (11).

The situation remains the same even if we consider the contribution to $\eta-\pi_0$ transition from other vector octet states.

Therefore the assumption of the $\eta-\pi_0$ transition to be dominant leads to the unique choice that the higher mass contribution belongs to a vector singlet.

Now the $\eta-\pi_0$ transition through ρ, ω and this intermediate states is given by

$$(13)\qquad M_{\eta\pi}(q^2) = \frac{e^2 f^2_{\pi\rho\omega}}{4\sqrt{3}\,\gamma_\omega \gamma_\rho}\left[\left(-\xi' + \xi\right) I_1(q^2) + \left(-\xi'\xi\right)\left(\frac{\gamma_\chi}{\gamma_\omega}\right)^2 I_2(q^2)\right] \Big/ m_\pi^2$$

where

$$I_1(q^2) = \frac{4}{(2\pi)^4}\int \frac{m_\rho^2}{m_\rho^2+k^2}\,\frac{m_\chi^2}{m_\chi^2+k^2}\,\frac{1}{k^2}\,\frac{k^2q^2-(kq)^2}{m_\rho^2+(q-k)^2}\,d^4k$$

and

$$I_2(q^2) = \frac{4}{(2\pi)^4}\int \left(\frac{m_\rho^2}{m_\rho^2+k^2}\right)^2 \frac{1}{k^2}\,\frac{k^2q^2-(kq)^2}{m_\chi^2+(q-k)^2}\,d^4k$$

These are convergent integrations and thus we have the following formulas which are valid for $0 < -q^2 \lesssim m_\eta^2$,

$$(14)\qquad \begin{aligned} I_1(q^2) &= \frac{1}{(4\pi)^2}\,\frac{1}{6}\,\frac{3}{4}\,q^2\,\frac{m_\chi^2}{3}\ , \\ I_2(q^2) &= \frac{1}{(4\pi)^2}\,\frac{1}{6}\,\frac{3}{4}\,q^2\,\frac{m_\rho^2}{4}\ . \end{aligned}$$

We see here that the ratio of the second term with respect to the first in eq. (13) is really small, for m_X $2m_p$ and $\xi = -\xi' = -1.12$, that is, $\sim \frac{1}{16}\left(\frac{\gamma_X}{\gamma_\omega}\right)^2$. Although we do not know the value γ_X, it may not differ much from γ_ω and then we may disregard the second term in eq. (13).

Thus we get the $\eta-\pi$ transition coefficient

$$
\ell_{\eta\pi} = M_{\eta\pi}(q^2)\frac{1}{q^2+m_\pi^2}\Big|_{q^2=-m_\eta^2} = \frac{\alpha}{\pi}\,0.86(\xi'-\xi) = 0.61\,\alpha \quad \text{for } \xi'=-\xi=1.12 \tag{15}
$$

$$
= 0.7\,\alpha \quad \text{for } \xi'=1.5,\ \xi=-1.12
$$

Here let us define an effective Hamiltonian density for the S-wave $\pi-\pi$ scattering by eq. (16),

$$
H_{\pi\pi} = \frac{d}{8}(\vec{\pi}\,\vec{\pi})(\vec{\pi}\,\vec{\pi}), \tag{16}
$$

Then the decay rate for $\eta \to \pi^+\pi^-\pi^0$ is

$$
P(\eta\to\pi^+\pi^-\pi^0) = \left(\frac{\sqrt{6}\,d}{4\pi}\right)^2 \ell_{\eta\pi}^2\ 0.020\ m_\pi \tag{17}
$$

Unfortunately we are not in a position to have a definite value of the scattering length for T = 0, S-wave $\pi-\pi$ scattering. So we shall estimate P($\eta\to\pi^+\pi^-\pi^0$) for several choices of the scattering length, ie, $a_{s_0} = 0.5$[6], 1, 1.5[6] m_π^{-1}. These correspond to the following values of the effective coupling constants: $d^*/4\pi = 0.4,\ 0.8,\ 1.2$.

The decay rate for $\eta\to\pi^+\pi^-\pi^0$ and the branching ratio P($\eta\to 2\gamma$)/P($\eta\to\pi^+\pi^-\pi^0$) estimated in this way are given in the table I.

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* In the lowest order perturbation approximation, $d/4\pi = 4\lambda$ where λ is given by Chew and Mandelstam[8].

TABLE I

	$\xi' = -\xi = 1.12$			$\xi' = 1.5, \quad \xi = -1.12$		
$-d/4\pi$ $(m_\pi A_{S_0})$	0.4 (0.5)	0.8 (1)	1.2 (1.5)	0.4 (0.5)	0.8 (1)	1.2 (1.5)
$\Gamma(\eta\to\pi^+\pi^-\pi^0)$ in 10^{-4} MeV	0.53	2.13	4.8	0.7	2.9	6.5
$\gamma_1^{(*)} \equiv \frac{\Gamma(\eta\to 2\gamma)}{\Gamma(\eta\to\pi^+\pi^-\pi^0)}$	1.2	0.3	0.13	15.0	3.7	1.7

(*) The linear dependence of the $\eta \to 3\pi$ matrix element on y, which is an experimental fact for the Dalitz plot of the $\eta \to 3\pi$, tells us that the ratio $\Gamma(\eta \to \pi^0\pi^0\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$ is 1.5 in the non-relativistic limit.

The evaluation of the decay rate $P(\eta \to \pi^+\pi^-\gamma)$ is carried out through a ρ meson pole diagram. As the vector singlet state x gives no contribution to this process, we get the ratio,

$$(18) \quad \gamma_2 \equiv \frac{P(\eta \to \pi^+\pi^-\gamma)}{P(\eta \to 2\gamma)} = \frac{\gamma_\rho^2}{4\pi}\, \frac{\gamma_{\rho\pi\pi}^2}{4\pi}\, \frac{1}{\alpha}\, \frac{1.0 \times 10^{-3}}{[1-\frac{1}{3}(1+2\xi')]^2}$$

For $\xi' = 1.12$ and $\xi' = 1.5$ with $\gamma_\rho^2/4\pi \sim \gamma_{\rho\pi\pi}^2/4\pi \sim 1/2$, γ_2 takes the values 5.3 and 0.3 respectively. Thus we see that the case of exact unitary symmetry is not consistent with the observed data (9) whereas the second case, with small deviation from unitary symmetry, is not necessarily inconsistent.

4) We started with the assumption that the vector particle γ transition is important whenever the e.m. interaction comes in. The observed form factor for $\pi^0 \to \gamma e^+ e^-$, then, tells us that there should be some contribution from the higher mass states besides those from ρ and ω. We represented them by vector particle states. Here we have two choices for taking them as the octet states and as the singlet state.

Both cases lead to the cancellation for the $\pi^0 \to 2\gamma$ amplitude and then the $\eta \to 2\gamma$ suppression follows from this. This gives the possibility of explaining the large discrepancy for the branching ratio γ_1 between the simple calculation and the experiment.

A difference, however, appears between octet states and a singlet state, if we consider the $\eta - \pi$ and the

$\eta \to \pi^+\pi^-\gamma$ transitions. The vector octet gives no $\eta - \pi$ transition and suppresses $\eta \to \pi^+\pi^-\gamma$ while the vector singlet leads to a large $\eta - \pi$ transition coefficient and to no cancellation for $\eta \to \pi^+\pi^-\gamma$.

We made an assumption that the $\eta - \pi$ transition is important. In this way we can examine if this assumption is consistent with other observed data, as well as the validity of a choice for a vector singlet state.

As a matter of fact, the results which we get in this case depend on the value of the scattering length for the S - wave $\pi - \pi$ scattering. For example, if we consider a small deviation from unitary symmetry and take $\xi' = 1.5$, $\xi = 1.12$, the branching ratios γ_1, γ_2, given in the table I and by eq. (18) for $a_{S_0} = 1.5\ m_\pi^{-1}$, may not be in disagreement with the experiment.

But if a_{S_0} turns out to be as small as $1\ m_\pi^{-1}$ or the experimental value for γ_2 settles smaller than 0.1, it seems difficult to explain both γ_1 and γ_2 by this model. In other words we have to give up with the $\eta - \pi$ transition mechanism as well as with the choice of the vector singlet.

The other test for this model will come from the decay rate for $w \to 3\pi$. With a vector singlet, as it does not contribute to this process, we get $\Gamma(\omega \to 3\pi) \sim 10$ MeV. While with the vector octet, where a cancellation happens, we have still the possibility of getting a smaller value for this.

As regards the $\eta - 3\pi$ decay in this case the alternative to the $\eta - \pi$ transition will be the diagram shown in Fig. 2, although we have to encounter an unknown coupling

$f_{\varphi\rho\rho}$, where φ is a T = 0, 0^{++} particle [10], and the result is more ambiguous.

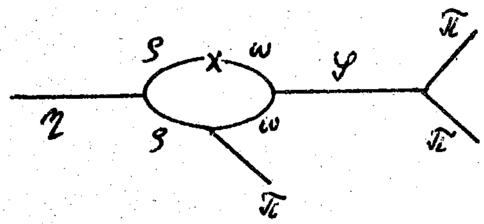

FIG. 2

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